AIAA 82-4011

Underexpanded Free Jets and their Interaction with Adjacent Surfaces

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Nomenclature

d	= diameter
M	= Mach number
Re_c	$= \rho_c a_c d_c / \mu_c$, nozzle throat Reynolds number
r	= radius
r, θ	= polar coordinates (Fig. 2)
α_e	= nozzle half angle
γ	= specific heat ratio
$\nu(M)$	= Prandtl-Meyer angle
ρ	= density
θ_∞	$= \nu(M \to \infty) - \nu(M_e) + \alpha_e$, direction of jet outermost streamline

Subscripts

0	= nozzle stagnation chamber
c	= nozzle throat
e	= nozzle exit section
∞	= ambient gas

Abstract

THE first part of this paper describes an underexpanded jet exhausting from a sonic or supersonic nozzle. Density distribution within the flowfield and jet size are given as approximate semiempirical expressions. The second part of the paper concerns the pressure distribution on a flat plate impinged upon by an underexpanded jet. Free jets interacting or not with surfaces are widely used in fundamental research and they frequently create problems in the design of space vehicles. This paper is intended to facilitate rapid estimations of both the characteristics of a jet and the dynamic effects of its impingement upon a wall.

Contents

A description of an underexpanded jet exhausting into a vacuum is given in Ref. 1. Based on this description, the density distribution in the jet may be rewritten as

$$\rho/\rho_0 = J_1 f(\theta) / [2I(r/r_c)^2] \quad (1)$$

J_1 is a function of γ which may be approximated by

$$J_1 = 0.538[(\gamma-1)/2]^{0.457} \quad (2)$$

The angular density distribution $f(\theta)$ is approximated by an empirical fit to numerical results[2]

$$f(\theta) = \{\cos[(\pi/2)(\theta/\theta_\infty)]\}^{2/\gamma-1} \quad (3)$$

The parameter I is a function of γ and θ_∞, where θ_∞ is the direction of the outermost streamline of the jet. The influence of M_e and α_e is included in I. The numerical integration required to calculate I leads to results closely fitted by

$$I = 0.613[(\gamma-1)/2]^{0.835}[\theta_\infty/(\pi/2)]^{1.92} \quad (4)$$

Substituting Eqs. (2) and (4) into Eq. (1) yields

$$\rho/\rho_0 = 0.439[(\gamma-1)/2]^{-0.378}[\theta_\infty/(\pi/2)]^{-1.92}(r/r_c)^{-2} f(\theta) \quad (5)$$

Equation (5) is based on the hypothesis of hypersonic isentropic flow from a virtual source located in the center of the nozzle exit section. Moreover, if Eq. (3) is used for $f(\theta)$, its validity is restricted to that part of the flow issued from the isentropic core of the nozzle. For a jet exhausting into an external gas at nonzero pressure P_∞, Eq. (5) holds in the region limited by the jet lateral shocks and the Mach disk. It is more general than the correlation of Ref. 3 and appears to be more correct than that of Ref. 4, when compared to a method-of-characteristics solution.

Using the above description, together with the hypothesis that the static pressure just downstream of the Mach disk is equal to P_∞, Mach disk distance from the nozzle r_{MD} is obtained as $(r_{MD}/r_c)^2 = [J_2/(2I)][P_0/P_\infty]$ where J_2 may be approximated by $0.752[(\gamma-1)/2]^{-0.428}$.

Substituting the expressions of J_2 and I yields

$$(r_{MD}/r_c)^2 = 0.619[(\gamma-1)/2]^{-1.263} \times [\theta_\infty/(\pi/2)]^{-1.92}[P_0/P_\infty] \quad (6)$$

Equation (6) is in reasonable agreement with previous successful correlations at low values of M_e and is thought to be more valuable at higher M_e because it is derived by physical considerations from a satisfactory model of the jet.

Introducing a reference length L_{ref} equal to Mach disk distance r_{MD} and a reference density ρ_{ref} equal to the density just ahead of the Mach disk, the density distribution on the axis of the jet is

$$\rho/\rho_{ref} = (r/L_{ref})^{-2} \quad (7)$$

with $\rho_{ref}/\rho_0 = J_3 P_\infty/P_0$ and $J_3 = (\gamma^2-1)/4\gamma \simeq 0.709 \; [(\gamma-1)/2]^{0.885}$. Experimental axial density distributions[1,5] obtained for $2.28 \leq M_e \leq 6.53$, $1.3\times10^4 \leq P_0/P_\infty \leq 6.6\times10^5$, $0 \leq \alpha_e \leq 30$ deg, and $2\times10^3 \leq Re_c \leq 4.6\times10^4$ have been plotted in Fig. 1. Departures from the "universal" curve given by Eq. (7) occur essentially at a low abscissa ($r/r_e < 5$) and near the expected Mach disk location.

An approximate method for the evaluation of pressure distribution on a flat plate located at distance h from the nozzle (Fig. 2) consists in calculating a local pressure by the Newtonian pressure concept, under the hypothesis that the jet remains undisturbed until it reaches the plate itself.[6] Using

Received Sept. 10, 1980; synoptic received June 29, 1981. Full paper available from National Technical Information Service, Springfield, Va. 22151 at the standard price (available upon request).

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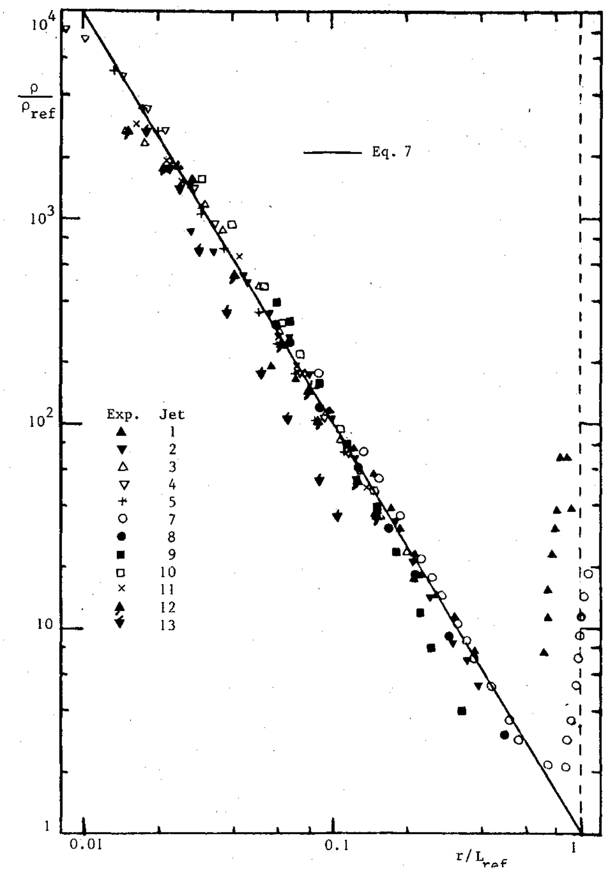

Fig. 1 Axial density profiles.

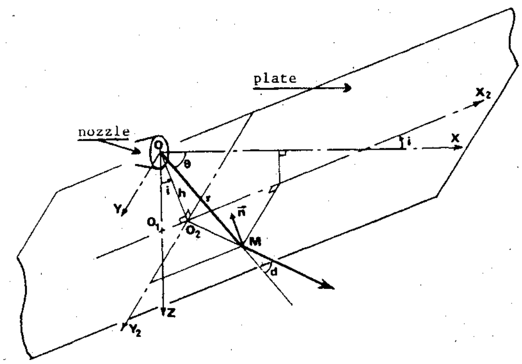

Fig. 2 Jet-plate interaction: definition of geometrical variables.

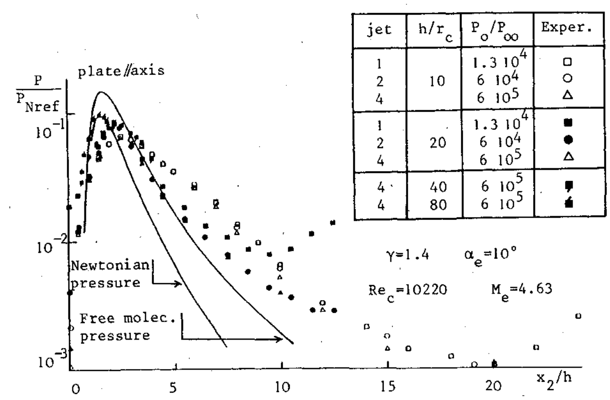

Fig. 3 Pressure distribution: influence of pressure ratio and wall distance.

the above description of the undisturbed jet and assuming P_∞ to be zero (or sufficiently low for the dimensions of the jet to be much longer than h) leads to the following expression for the Newtonian pressure P_N

$$P_N/P_{N\text{ref}} = (h/r)^4 f(\theta)/(2I) \tag{8}$$

where r and θ are the polar coordinates of the point under consideration and are easily expressed in terms of the Cartesian coordinates x_2 and y_2 on the plate. $P_{N\text{ref}}$ is a reference pressure defined as $P_0 J_4 (r_c/h)^2$ and J_4 may be approximated by $0.978\,[(\gamma-1)/2]^{-0.337}$.

Typical results obtained by the authors for the pressure distribution along the x_2 axis of the plate have been plotted in Fig. 3 together with the prediction of Eq. (8) and another prediction based on a free molecular formulation in place of the Newtonian approximation. The experimental profiles exhibit a rise at large abscissas when the Mach disk is approached.

Simple formulas have been derived for density distribution in an underexpanded jet for Mach disk distance and for pressure distribution on a plate impinged upon by a jet. Reference quantities have been introduced, which include the influence of most parameters (M_e, α_e, γ, h, and P_0/P_∞) so that the resulting expressions involve only a limited number of nondimensionalized variables. This facilitates comparisons between experimental as well as theoretical results and enables the designer to use the results obtained under certain conditions for the solution of problems involving other conditions. Predictions for density distribution and Mach disk distance agree satisfactorily with experiment. Predictions for pressure distribution give only the correct order of magnitude and qualitative information on the influence of parameters. Experiment demonstrates that $P_{N\text{ref}}$ and h are adequate scaling lengths and that the nondimensionalized pressure distribution on the plate depends little on M_e and α_e. More results, as well as a more detailed discussion, are given in the backup document.

Acknowledgment

This work was partially supported by Centre National d'Etudes Spatiales and Direction des Recherches et Etudes Techniques.

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